Relaxation of the turbulent magnetosheath

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In turbulence, nonlinear terms drive energy transfer from large-scale eddies into small scales through the so-called energy cascade. Turbulence often relaxes toward states that minimize energy; typically these states are considered globally. However, turbulence can also relax toward local quasi-equilibrium states, creating patches or cells where the magnitude of nonlinearity is reduced and energy cascade is impaired. We show, for the first time, compelling observational evidence that this "cellularization" of turbulence can occur due to local relaxation in a strongly turbulent natural environment such as the Earth's magnetosheath.

Introduction.–Turbulence is disordered, but may also involve relaxation processes that drive the system toward statistically predictable states [1]. The problem of turbulent relaxation has been addressed from theoretical [2–4] and numerical [5–7] perspectives. Relaxation may lead to the emergence of long-time metastable quasi-equilibrium states. However, there is evidence that [8–12] partially relaxed states can emerge on time scales on the order of a large-scale nonlinear (eddy turnover) times. Rapid local relaxation principles can contribute to the understanding of nonlinear processes in space plasma, astrophysical and geophysical systems, by accounting for the generation of distinctive measurable correlations. Here we present observational evidence that several forms of rapid relaxation – including Alfvénic, Beltrami, and force-free states – occur in the highly turbulent terrestrial magnetosheath plasma.

A canonical example is two-dimensional (2D) hydrodynamics, which exhibits both slow emergence of metastable global maximum entropy states [13, 14] as well as a local multiscale rapid relaxation [15]. This system is relevant to laboratory plasmas [16] geophysical flows, ionospheric structure [17], and other self-organizing systems [18].

Two classes of relaxation have been discussed in magnetoydrodynamics (MHD): (I) Selective decay processes [3, 19] in which a global long-wavelength relaxed state emerges by minimization of one ideal invariant (e.g., energy) relative to another (e.g., magnetic helicity) [20]; an example is Taylor relaxation [21, 22]; and (II) dynamic alignment processes, in which a distinctive correlation, such as large amplitude Alfvénic fluctuations, emerge dynamically [23, 24]. Both types reduce the amplitude of nonlinearities [7].

Plasma relaxation is also relevant to solar and interplanetary studies, where for example, magnetic clouds are typically modeled as force-free Lundquist states [25]. Taylor relaxation may drive these somewhat isolated magnetic flux tubes toward force-free states [26]. A broader class of target Grad-Shafranov equilibria are force-balance states, [27], which can be identified in solar wind data [28, 29]. Maximum entropy states [30] are also purported to emerge through turbulent relaxation.

Space physics studies of relaxation have previously identified [31], relaxed Alfvénic patches of turbulence at 1 au. Moreover, the "patchiness" of dynamically aligned states supports the idea of cellularized structure of magnetohydrodynamics (MHD) turbulence [32], the appearance of intermittency, as well as the incompatibility with the idea of turbulence arising from a superposition of Gaussian fields. The above-mentioned works investigating turbulence in the solar wind were necessarily limited by the lack of in-situ multi-spacecraft missions. Therefore, their analyses could investigate only the field alignments that do not involve the computation of derivatives e.g. Alfvénic states which require an (anti)alignment between the velocity and magnetic fields. Preliminary work to quantify these correlations has been done using Cluster solar wind data [33].

The present paper closes a gap left in the past, namely the lack of experimental evidence assessing the potential alignments predicted by the MHD theory. This is made possible by employing data from the Magnetospheric Multiscale (MMS) Mission [34]. In addition, turbulence in the Earth's magnetosheath can be imagined as "young" [35] since it is freshly modified by the solar wind passing through the bow shock. Identifying relaxed states in the magnetosheath strongly suggests that such states emerge quite rapidly.

Relaxation processes in MHD.–The relaxation processes under consideration can be studied in the context of simple MHD model equations that consist of a mix of linear and nonlinear terms. It is straightforward to show that states of minimum energy coincide with a minimization of the magnitude of the nonlinear terms. Ideal incompressible MHD equations read $\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\,\boldsymbol{v} = -\boldsymbol{\nabla} p + \boldsymbol{j}\times\boldsymbol{b}$, $\frac{\partial \boldsymbol{b}}{\partial t} = \boldsymbol{\nabla}\times(\boldsymbol{v}\times\boldsymbol{b})$, together with the constraints $\boldsymbol{\nabla}\cdot\boldsymbol{v} = \boldsymbol{\nabla}\cdot\boldsymbol{b} = 0$. In these equations, $\boldsymbol{a}$ is the vector potential such that $\boldsymbol{b} = \boldsymbol{\nabla}\times\boldsymbol{a}$, $\boldsymbol{j} = \boldsymbol{\nabla}\times\boldsymbol{b}$ is the current density, p is the kinetic pressure and $\boldsymbol{v}$ is the velocity field.

The three quadratic rugged invariants of 3D-MHD [2, 36] are the total energy $E = \frac{1}{2}\int_V \left(|\boldsymbol{v}|^2 + |\boldsymbol{b}|^2\right) d^3x$, and the two pseudo-scalars: the cross helicity $H_c = \frac{1}{2}\int_V \boldsymbol{v}\cdot\boldsymbol{b}\ d^3x$, and the magnetic helicity $H_m = \frac{1}{2}\int_V \boldsymbol{a}\cdot\boldsymbol{b}\ d^3x$, with the integrals being evaluated over an appropriate volume V.

The states that minimize energy keeping H_c and H_m constant can be recovered solving the variational problem $\delta\int\left(|\boldsymbol{v}|^2 + |\boldsymbol{b}|^2 - 2\alpha\,\boldsymbol{v}\cdot\boldsymbol{b} + 2\phi\,\boldsymbol{a}\cdot\boldsymbol{b}\right) d^3x = 0$ where 2α and 2ϕ are Lagrange multipliers. The obtained constrained fields are such that $\boldsymbol{v} = \alpha\boldsymbol{b} = \frac{\alpha(1-\alpha)^2}{\phi}\boldsymbol{j} = \frac{(1-\alpha)^2}{\phi}\boldsymbol{\omega}$ where $\boldsymbol{\omega} = \boldsymbol{\nabla}\times\boldsymbol{v}$ is the vorticity [7, 10, 33].

One immediately notices that the above relationships describe fields that tend to suppress the non-linear terms in the MHD equations. They are also associated with particular equilibrium states, such as the Taylor force-free state $\boldsymbol{j}\propto\boldsymbol{b}$ [21, 37]. However, the reduction of nonlinearities and incomplete relaxation to the target states is of primary interest here. Moreover, suppression of nonlinearity is even more widespread; if one rewrites the term $(\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v} = \boldsymbol{\nabla}\left(\frac{v^2}{2}\right) - \boldsymbol{v}\times\boldsymbol{\omega}$, it will be shown below that the terms on r.h.s. also evolve toward (anti)alignment. In addition, the Lorentz force term $(\boldsymbol{j}\times\boldsymbol{b})$ and the Lamb vector $(\boldsymbol{v}\times\boldsymbol{\omega})$, each contributing to accelerations, tend to anti-align, thus further suppressing any residual nonlinearity.

A convenient measure for identifying (local) partially relaxed states is the inner product $(\,,\,)$. Given two vectors $\boldsymbol{f}$ and $\boldsymbol{g}$, we evaluate the distribution of the cosines of the comprised angles, namely: $\cos(\theta) = \frac{(\boldsymbol{f},\boldsymbol{g})}{||\boldsymbol{f}||\,||\boldsymbol{g}||}$ where $||\cdot||$ represents a vector magnitude. When $\boldsymbol{f}$ and $\boldsymbol{g}$ are (anti)aligned 3D vectors, the distribution of the cosine of the angles between the two tends to peak at values $\cos(\theta) = \pm 1$. On the other hand, if they have random orientations with respect to one another, the distribution will be flat at $\cos(\theta)\sim 0.5$.

Data selection and analysis.–We use a total of 1180 MMS intervals in the turbulent magnetosheath from 2015 Sep 08 through 2018 Jan 01. These range in duration from 50 to 540 seconds as reported in Fig. 1. Velocity and magnetic field measurements are obtained from the Fast Plasma Investigation [FPI, 38] and the Fluxgate magnetometers [FGM, 39], respectively. Magnetic field burst measurements (128 Hz) have been resampled to match the resolution of ion bulk velocity (150 ms). Spacecraft spintone is removed from the ion data.

As a check on the FPI data quality, we found density values greater than 50cm^{-3}, which might be less reliable. 172 intervals out of 1180 ($\lesssim 15\%$) have an average number density $> 50\text{cm}^{-3}$. The subsequent analysis will not be greatly affected by these intervals.

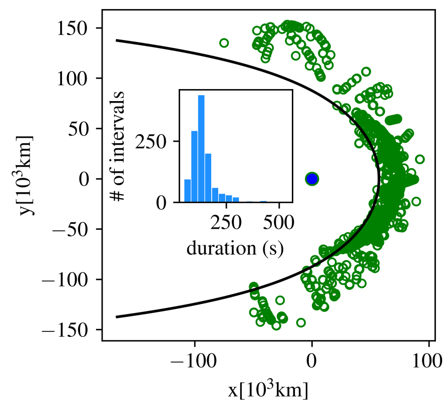

FIG. 1. Nominal position of the Earth's magnetopause in GSE coordinates according to [40] (black curve), and locations of the MMS intervals analyzed (green circles). Earth is indicated as a blue circle. The inset shows the histogram of the durations of such intervals.

The vorticity and current density fields are computed using the reciprocal vector technique for tetrahedra [41], resulting in a single vector time series for each. We then interpolate the bulk flow speed and the magnetic field from the four MMSs to the barycenter of the tetrahedron. Before computing the alignment cosines, it is necessary to low-pass filter all the fields to focus on inertial-range scales [42]. This is done by averaging over a 1-second running window, without changing the resolution. At last, the mean values are subtracted from the bulk flow and the magnetic field, since alignments are calculated between fluctuations.

Results.–For each interval, the cosine of the angles between all the appropriate pairs of fields has been measured. As one would expect, not all intervals show indications of alignment between pairs of fields; some intervals show a good degree of relaxation, while others do not. Statistical significance is obtained by compiling distributions of the cosines of the angles of each type from all 1180 intervals used in the survey.

Figure 2 shows the distribution of the alignment cosines for several relevant pairs of fields. Each pair, when aligned (or, anti-aligned) suppresses a nonlinear term in the dynamical equations. It is apparent that there are varying degrees of alignment, and therefore varying degrees of suppression. We now briefly comment on these.

The *Alfvénic state*, i.e., pointwise $(\boldsymbol{v},\boldsymbol{b})$ alignment, is one of the most fundamental and frequently discussed states being associated both with linear MHD Theory and nonlinear theory. When MHD equations are written in the Elsässer variables (defined in terms of velocity

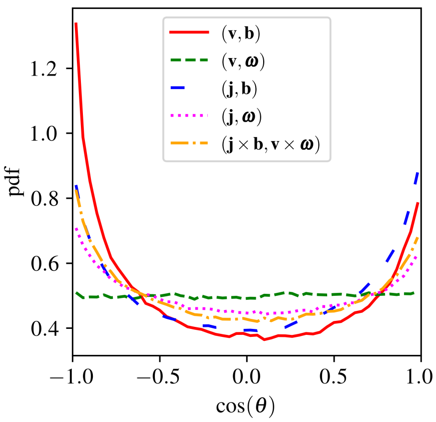

FIG. 2. Global distributions of alignments evaluated over all selected intervals.

and magnetic field fluctuations $\boldsymbol{z}^{\pm} = \boldsymbol{v} \pm \boldsymbol{b}$), it is evident that incompressible nonlinearity arises from the interaction between the $\boldsymbol{z}^{+}$ and $\boldsymbol{z}^{-}$ fields. When cross helicity is high the evolution of turbulence is hampered; this requires both $(\boldsymbol{v}, \boldsymbol{b})$ alignment, and energy equipartition between ($\boldsymbol{v}$ and $\boldsymbol{b}$). We also investigated the behavior of $(\boldsymbol{v}, \boldsymbol{b})$ alignment after sector-rectifying the magnetic field [31], but little change was seen to occur. Although the sign of cross helicity is crucial in determining whether the Alfvénic fluctuations propagate outward (anti-sunward) or inward, it has no effect on the alignment. The study of the propagation of Alfvénic fluctuations in the magnetosheath is a problem that needs to be addressed more thoroughly [43], but such a discussion is beyond the scope of this paper.

The velocity and vorticity fields do not display global Beltrami alignment, indicated in Fig. 2 by $(\boldsymbol{v}, \boldsymbol{\omega})$. The same result was found in 3D-MHD simulations [10] and attributed to the fact that kinetic helicity, $\int \boldsymbol{v} \cdot \boldsymbol{\omega}\, d^3 r$, is not an invariant and is therefore unconstrained. Beltrami-like alignments have also been previously discussed in solar wind observations [33] where accuracy is limited by large spacecraft separations.

The status of Beltrami $(\boldsymbol{v}, \boldsymbol{\omega})$ correlations differs in hydrodynamics (HD). A pioneering work, [8] studied velocity-vorticity patterns in turbulent flows and found that $\boldsymbol{v}$ and $\boldsymbol{\omega}$ align in the center of a channel where the flow is relatively laminar. The alignment disappears in the turbulent transition region between the walls and the central region. When only fluctuations, and not the mean flow, were considered, alignment was lost throughout the whole channel. This parallels the present result if one imagines flux ropes to be counterparts of HD channels. A second interesting interpretation follows from the correlation between the vorticity and the rate of strain tensor and its eigenvectors. Ref. [8] found that where the dissipation (measured as $S_{ij}^2 = 1/2(\partial u_i/\partial x_j + \partial u_j/\partial x_i)^2$) is large, the alignment is lost.

Enhancement of current-magnetic field alignment $(\boldsymbol{j}, \boldsymbol{b})$ implies an approach to a force-free condition, equivalent to the Beltrami property for the magnetic field ($\nabla \times \boldsymbol{b} \propto \boldsymbol{b}$). Such states lend naturally to a description in terms of flux tubes, which may be locally nearly force free, but also interacting, perhaps mainly at their boundaries [44]. When not force-free, such flux tubes may be nearly force balanced which represents an additional type of cellularization of turbulence. This type of suppression of nonlinearity, involving pressure balances, will be examined in a separate study. Here we do not involve pressure at this stage as it introduces complications such as considerate separate electron and proton pressures (separate plasma beta) [45]. Force-balanced states are often studied as Grad-Shafranov equilibria[27].

The $(\boldsymbol{j}, \boldsymbol{\omega})$ alignment could heuristically be related to the dynamics of the Alfvénic state, but it can also be linked to other basic properties of turbulence. In the equation that involve the generalized vorticity $\boldsymbol{\Omega}^{\pm} = \boldsymbol{j} \pm \boldsymbol{\omega}$ [46], the nonlinear term is $\boldsymbol{z}^{\pm} \cdot \boldsymbol{\nabla} \boldsymbol{\Omega}^{\mp}$ that is suppressed when $\boldsymbol{j} \sim \boldsymbol{\omega}$.

The Lorentz force and the Lamb vector show a slight anticorrelation as expected from the equations since these two terms should be of opposite signs in order to reduce any residual nonlinearity.

Transmission of relaxed patches across the bow shock.– How turbulence is processed crossing a shock is a subject of ongoing interest [47]. In particular, a recent work [48] examined the possibility that flux ropes can "survive" the crossing of the Earth's bow shock, with different degrees of deformations. We now demonstrate how relaxed Alfvénic patches of solar wind plasma are modified across the bow shock. The analysis focuses on the stream of plasma that was found to be sampled by both WIND in the pristine solar wind, upstream, and, 55 minutes later, by MMS downstream of the bow shock. The interval spans 2017 Dec 31 21:20 through 2018 Jan 01 01:20 UTC. Since Beltrami states cannot be measured by the single WIND spacecraft, we focus only on the Alfvénic state. We use measurements for velocity and magnetic fields from the Solar Wind Experiment [SWE, 49] and Magnetic Field Investigation [MFI, 50] suites on WIND. For consistency between the spacecraft, which have different time resolutions, we downsample the magnetic and velocity fields to a 1-minute cadence and then evaluate the angle between the two fields for WIND and MMS separately. The two distributions of the Alfvénic alignment are shown in Fig. 3.

Clearly, the presence of alignments in the pristine solar wind does not guarantee that these will persist downstream. The evidence suggests that a small fraction of alignment is maintained, with an overall significant reduction. There is some transmission across the bow shock, but the presence of relaxed patches in the magnetosheath may also be due to the relatively rapid turbulence evolution in the magnetosheath.

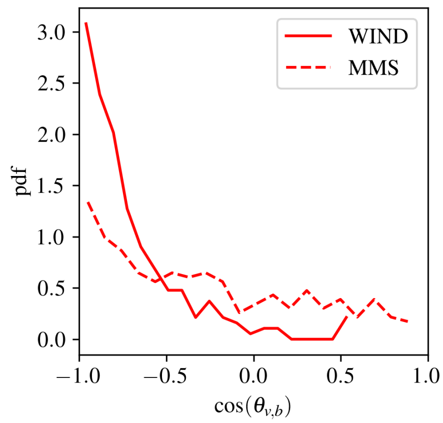

FIG. 3. Alfvénic alignment within the same plasma parcel measured by WIND and MMS, upstream and downstream of the Earth's bow shock respectively. The distributions indicate that shock crossing reduces the degree of alignment.

Residency times in relaxed patches.–Let us now investigate the link between the relaxed regions and the cellularization of turbulence. Starting from the cosine time series obtained as described in the previous sections, we measure for each interval how much time the system dwells above a certain alignment threshold (in absolute magnitude). The distributions of such residency times (RT) defined as cosine larger than 0.5 or smaller than −0.5 are shown in Fig. 4 and indicated on the left axis as pdf $\mathrm{RT}_{>0.5}$. The first feature one notices is that the Alfvén aligned states persist for longer times than the others, evidenced by a more extended distribution. The distributions show a more or less extended power law with a sharp cutoff at the upper end: This behavior is characteristic of self-similar phenomena, supporting the picture of cellularized turbulence comprising distinct patches of different sizes.

A standard estimate of the typical size of energy-containing eddies is given by the correlation scale. (Solar wind and magnetosheath measurements are often quoted in the time domain, the two being approximately proportional when invoking the Taylor frozen-in flow hypothesis [51].) Here the eddy size and correlation scale are estimated using the magnetic field autocorrelation function $R(\tau) = \langle \boldsymbol{b}(t) \cdot \boldsymbol{b}(t+\tau)\rangle_t$, where $\boldsymbol{b}$ are the magnetic field fluctuations and τ is the time lag. The average $\langle \dots \rangle_t$ is performed over the associated magnetosheath interval. The correlation time is determined as the first time lag at which the normalized autocorrelation function $\tilde{R}(\tau) = R(\tau)/R(0)$ falls to $1/e$ [e-folding, 52]. The histogram of the correlation times for each of our 1180 intervals is reported in Fig. 4.

Finally, for each residency time distribution, we calculate the first moment (frequency of occurrence) distributions (pdfs) $\mathcal{P}(*)$ as $T = \int \eta \, \mathcal{P}(\eta) \, d\eta$. The probability density functions are already normalized as $\int \mathcal{P}(\eta) \, d\eta = 1$. In Fig. 4 the first moments T for each pdf are indicated with vertical lines having the same line style as the respective probability density.

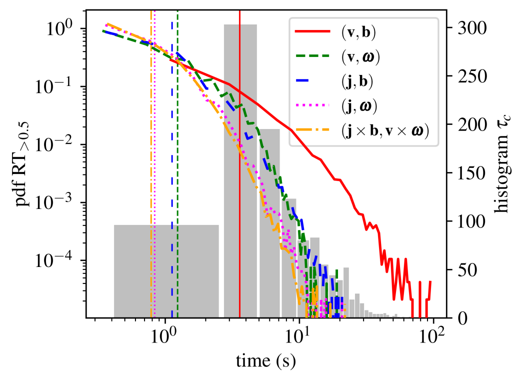

FIG. 4. Probability density functions of residency times in patches where the pairs of vectors have comprised angles with a cosine larger than 0.5 or smaller than −0.5 (left axis). Histogram of the correlation times calculated for all the considered intervals, gray bars (right axis). Vertical lines indicate first moments of respective pdfs.

The immediate feature one notices is the correspondence between the average residency time of the Alfvén state and the most probable correlation time. This correspondence is explained naturally by the above-mentioned cellularization of turbulence: The fabric of the turbulent solar wind is formed by patches (eddies) that tend toward a local equilibrium; their size can be estimated with the correlation time, and we have shown that it corresponds to the average time the magnetosheath plasma spends in an Alfvénic relaxed quasi-equilibrium.

The other classes of relaxed states persist, on average, for periods of time smaller than the Alfvénic states. There are several possible reasons for this: (i) non-Alfv'enic alignments involve derivatives of fields, therefore, acting on smaller, faster scales; (ii) the Alfvén state is more directly correlated with larger scale MHD phenomena, such as flux tubes. Additionally, the Beltrami and force-free states have similar time scales, suggesting an analogy between HD and MHD findings: From [8], the Beltrami state is found within the central channel flow region; likewise, while in plasma, the force-free region in a flux rope may be near the magnetic axis, far from the tube's borders, where discontinuities and bursts of activity, e.g., reconnection, are often found [44, 53, 54].

Discussion and Conclusions.– The rapid appearance of characteristic correlations due to relaxation processes have been addressed previously from theoretical [5], numerical [10], and observational [31] points of view. We presented here, for the first time in a natural plasma, a full study of several different contributions to relaxation and suppression of nonlinearity using MMS measurements in the magnetosheath.

Some of the special states discussed in this paper

(Alfvénic, force-free) are well-known and studied in several different aspects in the solar wind and plasma community; though others are not. In particular, the Beltrami state, well studied in hydrodynamics, is poorly addressed in MHD and solar wind studies. The possibility that the Beltrami state is attained where the dissipation is small and destroyed where it is large may set the direction for new studies correlating dissipation and relaxation in plasma. A fundamental measure of dissipation is the pressure work [55], which can be measured with MMS [56]. Ultimately we expect that intermittency of the dissipation may be anticorrelated with relaxation, as might be inferred from flux tube structure seen in both simulations [57] and observations [44, 58]. A clear direction for future research is to examine the spatial relationship between relaxation signatures and dissipation in greater detail.

Regarding the fast occurrence of relaxation, previous numerical works [10] assessed the emergence of alignments after a few Alfvén times. However, the related observational studies performed in the solar wind [31, 33] could not infer any temporal evolution, as 1 au turbulence potentially had at least several nonlinear/Alfvén times to evolve [35]. Conversely, the magnetosheath plasma consists of strongly perturbed solar wind plasma (by passage through the bow shock) while it also evolves on much more rapid timescales [59]. Therefore, the presence of relaxed patches in the magnetosheath suggests that relaxation is a fast-occurring phenomenon, but it is also possible that relaxed patches are degraded but still survive passage through the shock. It is worth mentioning the recent results from [48] that investigated the transmission of flux ropes across the Earth's bow shock. Their finding is that some structures can survive the crossing of the bow shock, hinting that relaxed patches downstream may come directly from the solar wind (possibly with a reduced degree of alignment.)

In hydrodynamics [46, 60, 61] relaxation often involves alignments of eigenvectors of the rate-of-strain tensor with other quantities, such as vorticity. Such alignments provide additional topological information. Such study, adapted to the turbulent fields in plasmas [62] may provide valuable physical insights in the study of magnetosheath turbulence.

By examining relaxation in the magnetosheath, this study closes a gap in the current literature. We also expect that it will motivate a number of additional studies as suggested above. A major extension to the solar wind awaits upcoming multispacecraft missions such as Helioswarm [63].

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